Using fractals to understand brain function

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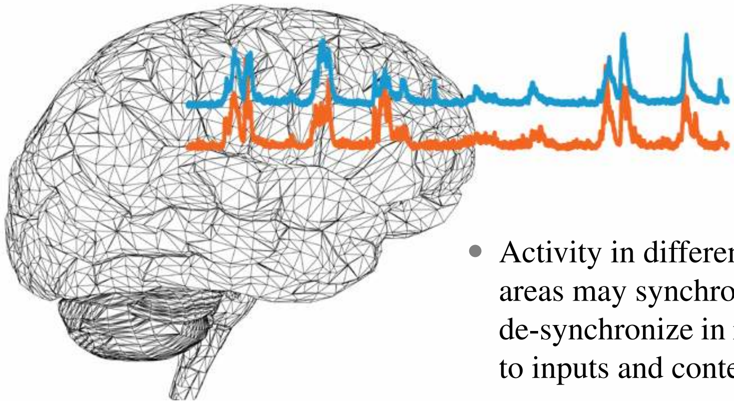
Mathematics and psychiatry

- Quantitative, physiology-based assessments are the gold standard in diagnosis and treatment of somatic conditions.
- In psychiatry, assessments are based primarily on statistical observations of behavior.
- That is because our quantitative, “mathematical” understanding of the brain has been limited.
- Progress needs to involves new mathematical ideas and techniques.

Synchronization in brain networks



- Activity in different brain areas may synchronize / de-synchronize in response to inputs and context.

- Brain regions communicate with each other via a complex network of connections.

Wikimedia Foundation. (2022)

Brain connectivity and dynamics

- Connecting pathways between brain regions (*nodes*) form the brain network (*connectome*).

- The network architecture underlies brain dynamical activity.

Brain connectivity and dynamics

- Brain dynamics control behavioral functions like emotion, cognition, movement, etc.

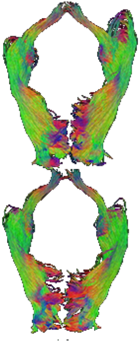
- Pathway impairment could be an underlying factor of abnormal behavior and mental illness.

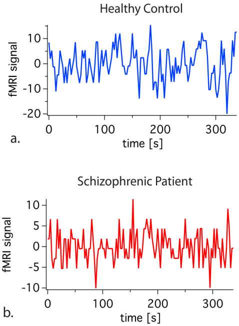
CONNECTIVITY → BRAIN DYNAMICS → BEHAVIOR

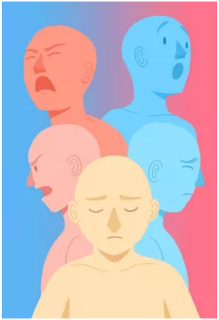
Brain connectome
(from DTI)

Dynamic signals
(from fMRI)

Outward symptoms
(observed)







USC's Mark and Mary Stevens Neuroimaging and Informatics Institute

Study objectives

- Conceive a simplified mathematical object to capture the relationship between brain connectivity and brain function.

- Use the geometry of this object to understand the ties between connectivity and function.

- Use it to predict brain function and behavior from connectivity information.

Mathematical approach

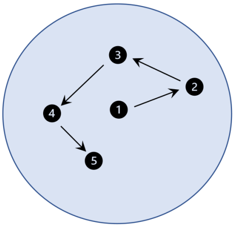
- Graph theory (to interpret the connectome).
- Discrete dynamics: repeated iterations of functions (to model the *node* dynamics).
- In combination: dynamic networks (or *dynomics*).

The Mandelbrot set

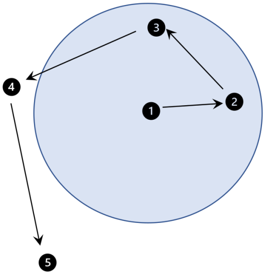
Family of quadratic functions: $f_c(z) = z^2 + c$.
Fix the parameter c in the plane.
Consider the map $f_c(z) = z^2 + c$.
Compute the *orbit* of $z = 0$, by repeatedly applying f_c.

$$0 \to f_c(0) \to f_c(f_c(0)) \to \dots$$

c is in the Mandelbrot set
if this sequence is bounded



c is not in the Mandelbrot set
if this sequence $\to \infty$



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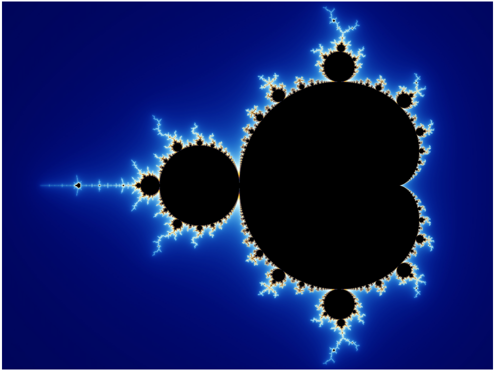
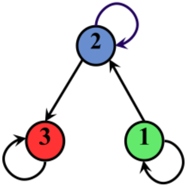


Mandelbrot sets for nodes in a network

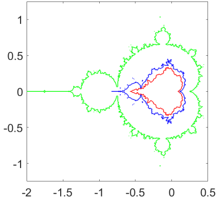
3D network



Connectivity matrix

$$\begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}$$

Mandelbrot sets

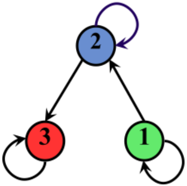


Defining equations:

$$\begin{aligned} z_1 &\rightarrow z_1^2 + c \\ z_2 &\rightarrow (z_1 + z_2)^2 + c \\ z_3 &\rightarrow (z_2 + z_3)^2 + c \end{aligned}$$

Mandelbrot sets for nodes in a network

3D network



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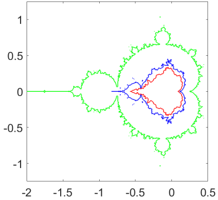


Example: $c = -2$ is in the $\mathcal{M}$ set for z_1, but not in the $\mathcal{M}$ set for z_2.

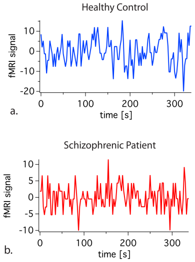
$z_1: 0 \to 0^2 - 2 = -2 \to (-2)^2 - 2 = 2 \to 2^2 - 2 = 2 \ldots$ (bounded)

$z_2: 0 \to (0+0)^2 - 2 = -2 \to (-2-2)^2 - 2 = 14 \to$
$\to (14+2)^2 - 2 \ldots$ (unbounded)

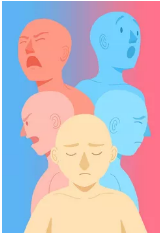
How we use mathematics

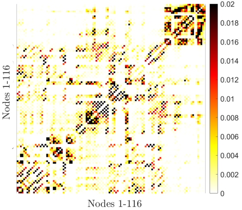
Connectome

Dynamics

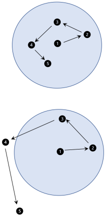


Behavior

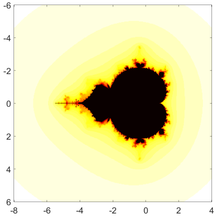




Matrix



Discrete dynamics



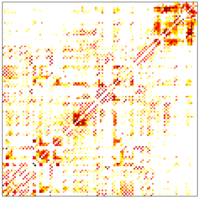
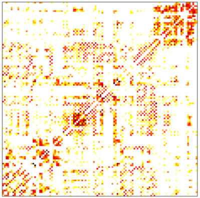
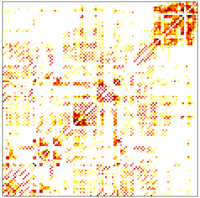
Mandelbrot sets

Data set used

- We tested our method on a set of tractography data for 197 individuals.
- The data was collected through the Human Connectome Project.
- The project aims to identify relationships between connectome patterns and behavior.
- **Brain dynamics** bridge connectome patterns to behavior. We use our dynamical systems approach to illustrate this.

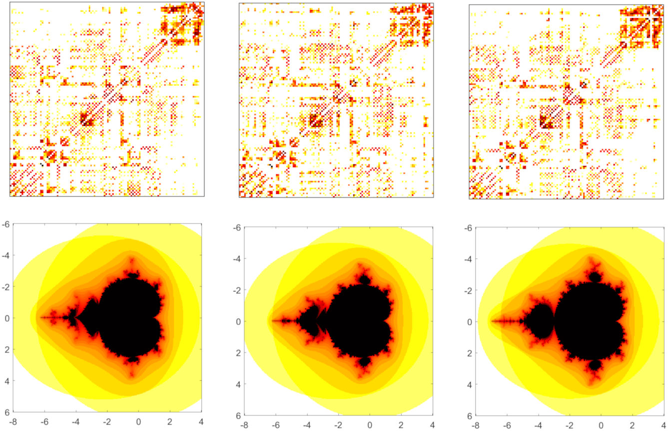
Application

- *"What is the role of weak connections in synchronization of brain activity?"*
- We extracted matrices from DTI connectomes (116 nodes).

Application

- For each participant, the corresponding Mandelbrot sets were computed (bottom).
- The geometry of the Mandelbrot sets is slightly different between participants, creating a *signature* for each individual.



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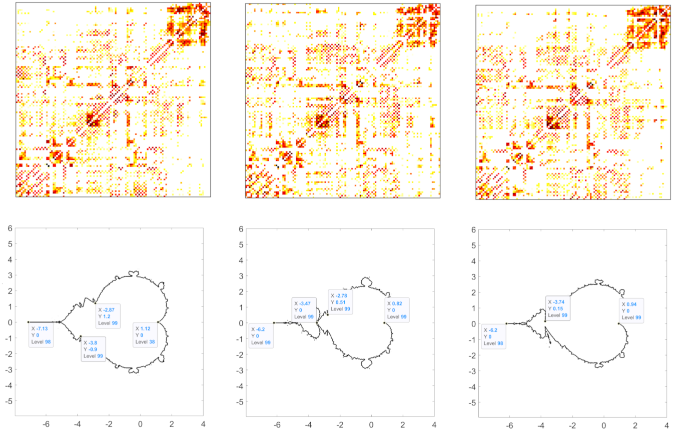
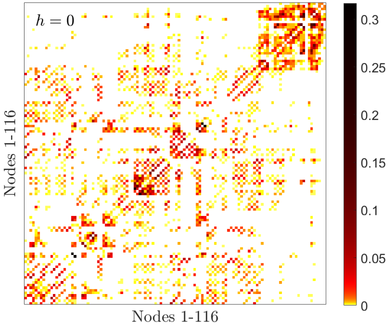
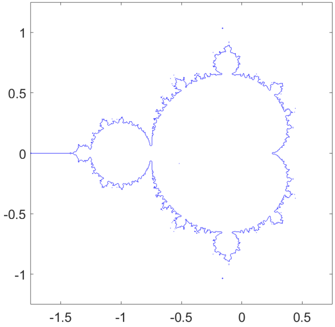


Results on synchronization

- Compute Mandelbrot sets separately for each node.
- Study when nodes *synchronize* (i.e., have identical $\mathcal{M}$).
- *"What is the role of weak connections in synchronization of brain activity?"*



100% connections



$K = 1$ contours

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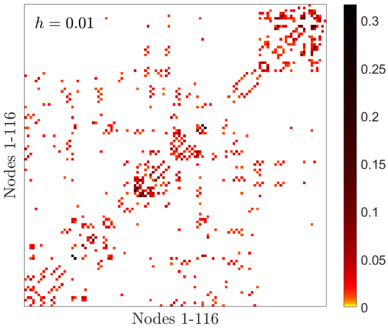
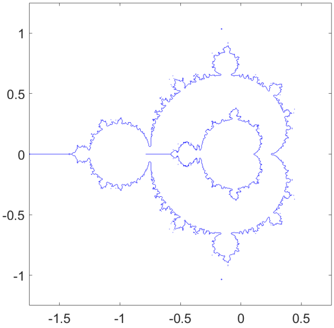


$\sim 20\%$ connections



$K = 2$ contours

Results on synchronization

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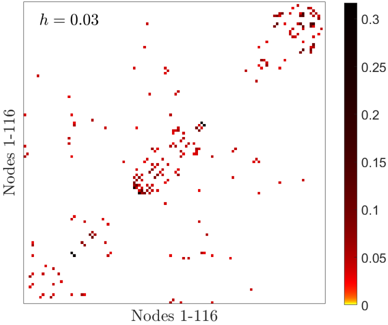
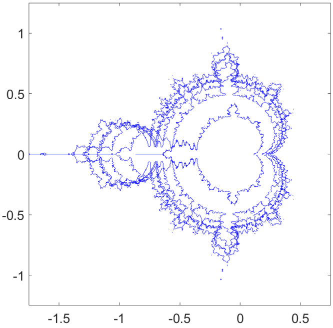


$\sim 7\%$ connections



$K = 6$ contours

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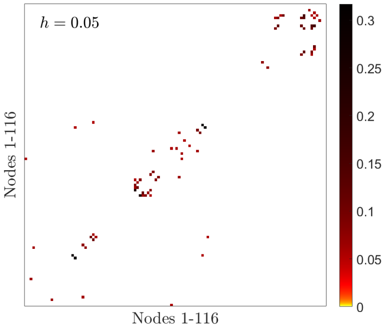
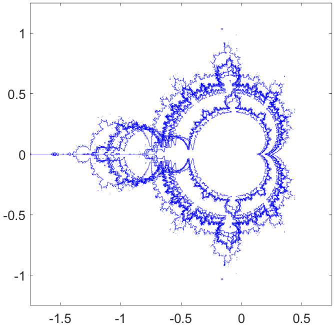


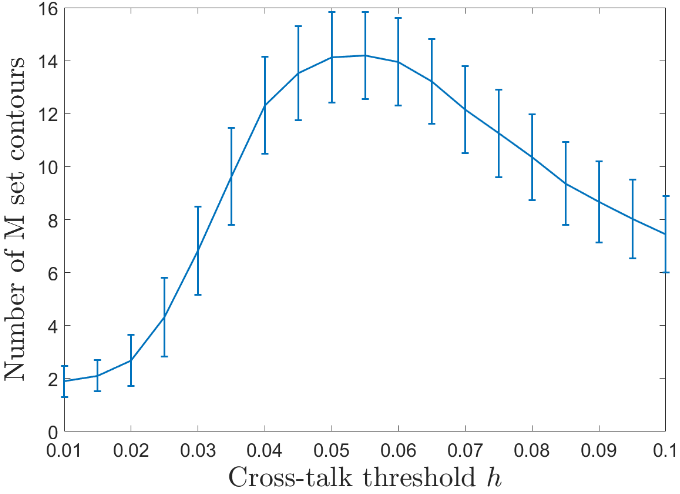


$\sim 3\%$ connections

$K = 12$ contours

Results on synchronization

Synchronization vs. cross-talk threshold (group statistics)



Conclusions

- Weak connections between brain regions play an extremely important role in synchronization.
- This has been observed before in networks, but has never been fully explained.
- The brain has to constantly switch between “synchronized” and “de-synchronized” patterns.
- Our mathematical approach showed that the brain can do this by simply tuning the weak connections.

Acknowledgements

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- Mentor: Dr. Anca Rădulescu.

References

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- A. Rădulescu, S. Evans, 2019. **J Complex Networks.** 7(3).
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Thank you!

Questions?