Using fractals to understand brain function

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Mathematics and psychiatry

• Quantitative, physiology-based assessments are the gold standard in diagnosis and treatment of somatic conditions.

• In psychiatry, assessments are based primarily on statistical observations of behavior.

• That is because our quantitative, “mathematical” understanding of the brain has been limited.

• Progress needs to involves new mathematical ideas and techniques.
Synchronization in brain networks

- Activity in different brain areas may synchronize / de-synchronize in response to inputs and context.

- Brain regions communicate with each other via a complex network of connections.

Wikimedia Foundation. (2022)
Brain connectivity and dynamics

- Connecting pathways between brain regions (nodes) form the brain network (connectome).

- The network architecture underlies brain dynamical activity.
Brain connectivity and dynamics

- Brain dynamics control behavioral functions like emotion, cognition, movement, etc.
- Pathway impairment could be an underlying factor of abnormal behavior and mental illness.
CONNECTIVITY → BRAIN DYNAMICS → BEHAVIOR

Brain connectome (from DTI)

Dynamic signals (from fMRI)

Outward symptoms (observed)

USC’s Mark and Mary Stevens Neuroimaging and Informatics Institute
Study objectives

• Conceive a simplified mathematical object to capture the relationship between brain connectivity and brain function.

• Use the geometry of this object to understand the ties between connectivity and function.

• Use it to predict brain function and behavior from connectivity information.
Mathematical approach

- Graph theory (to interpret the connectome).

- Discrete dynamics: repeated iterations of functions (to model the node dynamics).

- In combination: dynamic networks (or dynomics).
The Mandelbrot set

**Family of quadratic functions:** \( f_c(z) = z^2 + c \).

Fix the parameter \( c \) in the plane.

Consider the map \( f_c(z) = z^2 + c \).

Compute the orbit of \( z = 0 \), by repeatedly applying \( f_c \).

\[
0 \rightarrow f_c(0) \rightarrow f_c(f_c(0)) \rightarrow \ldots
\]

\( c \) is in the Mandelbrot set if this sequence is bounded

\( c \) is not in the Mandelbrot set if this sequence \( \rightarrow \infty \)
The Mandelbrot set

Family of quadratic functions: $f_c(z) = z^2 + c$.

Fix the parameter $c$ in the plane.
Consider the map $f_c(z) = z^2 + c$.
Compute the orbit of $z = 0$, by repeatedly applying $f_c$.

$$0 \rightarrow f_c(0) \rightarrow f_c(f_c(0)) \rightarrow \ldots$$
Mandelbrot sets for nodes in a network

3D network

Connectivity matrix

\[
\begin{pmatrix}
1 & 0 & 0 \\
1 & 1 & 0 \\
0 & 1 & 1
\end{pmatrix}
\]

Mandelbrot sets

Defining equations:

\[
\begin{align*}
\mathbf{z}_1 & \rightarrow \mathbf{z}_1^2 + c \\
\mathbf{z}_2 & \rightarrow (\mathbf{z}_1 + \mathbf{z}_2)^2 + c \\
\mathbf{z}_3 & \rightarrow (\mathbf{z}_2 + \mathbf{z}_3)^2 + c
\end{align*}
\]
Mandelbrot sets for nodes in a network

3D network

Connectivity matrix

\[
\begin{pmatrix}
1 & 0 & 0 \\
1 & 1 & 0 \\
0 & 1 & 1
\end{pmatrix}
\]

Mandelbrot sets

Example: \( c = -2 \) is in the \( \mathcal{M} \) set for \( z_1 \), but not in the \( \mathcal{M} \) set for \( z_2 \).

\( z_1: 0 \rightarrow 0^2 - 2 = -2 \rightarrow (-2)^2 - 2 = 2 \rightarrow 2^2 - 2 = 2 \ldots \) (bounded)

\( z_2: 0 \rightarrow (0 + 0)^2 - 2 = -2 \rightarrow (-2 - 2)^2 - 2 = 14 \rightarrow (14 + 2)^2 - 2 \ldots \) (unbounded)
How we use mathematics

**Connectome**

**Dynamics**

- Healthy Control
- Schizophrenic Patient

**Behavior**

**Matrix**

**Discrete dynamics**

**Mandelbrot sets**
Data set used

- We tested our method on a set of tractography data for 197 individuals.

- The data was collected through the Human Connectome Project.

- The project aims to identify relationships between connectome patterns and behavior.

- **Brain dynamics** bridge connectome patterns to behavior. We use our dynamical systems approach to illustrate this.
Application

• “What is the role of weak connections in synchronization of brain activity?”

• We extracted matrices from DTI connectomes (116 nodes).
Application

- For each participant, the corresponding Mandelbrot sets were computed (bottom).
- The geometry of the Mandelbrot sets is slightly different between participants, creating a *signature* for each individual.
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• For each participant, the corresponding Mandelbrot sets were computed (bottom).
• The geometry of the Mandelbrot sets is slightly different between subjects, creating a *signature* for each individual.
Results on synchronization

- Compute Mandelbrot sets separately for each node.
- Study when nodes synchronize (i.e., have identical $\mathcal{M}$).
- “What is the role of weak connections in synchronization of brain activity?”

100% connections

$K = 1$ contours
Results on synchronization

- Compute Mandelbrot sets separately for each node.
- Study when nodes synchronize (i.e., have identical $\mathcal{M}$).
- “What is the role of weak connections in synchronization of brain activity?”

$\sim 20\%$ connections

$K = 2$ contours
Results on synchronization

- Compute Mandelbrot sets separately for each node.
- Study when nodes synchronize (i.e., have identical $\mathcal{M}$).
- “What is the role of weak connections in synchronization of brain activity?”

$\sim 7\%$ connections  \hspace{1cm} $K = 6$ contours
Results on synchronization

- Compute Mandelbrot sets separately for each node.
- Study when nodes synchronize (i.e., have identical $M$).
- “What is the role of weak connections in synchronization of brain activity?”

$\sim 3\%$ connections  \quad $K = 12$ contours
Results on synchronization

Synchronization vs. cross-talk threshold
(group statistics)
Conclusions

- Weak connections between brain regions play an extremely important role in synchronization.

- This has been observed before in networks, but has never been fully explained.

- The brain has to constantly switch between “synchronized” and “de-synchronized” patterns.

- Our mathematical approach showed that the brain can do this by simply tuning the weak connections.
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References

• A. Rădulescu, S. Evans, 2019. J Complex Networks. 7(3).
Thank you!

Questions?