Using fractals to understand brain function

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Mathematics and psychiatry

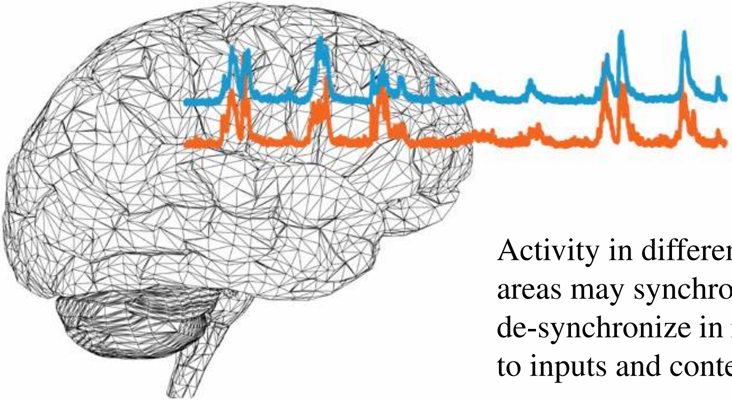
Quantitative, physiology-based assessments are the gold standard in diagnosis and treatment of somatic conditions.

In psychiatry, assessments are based primarily on statistical observations of behavior.

That is because our quantitative, “mathematical” understanding of the brain has been limited.

Progress needs to involves new mathematical ideas and techniques.

Synchronization in brain networks



Activity in different brain areas may synchronize / de-synchronize in response to inputs and context.

Brain regions communicate with each other via a complex network of connections.

Wikimedia Foundation. (2022)

Brain connectivity and dynamics

Connecting pathways between brain regions (*nodes*) form the brain network (*connectome*).

The network architecture underlies brain dynamical activity.

Brain connectivity and dynamics

Brain dynamics control behavioral functions like emotion, cognition, movement, etc.

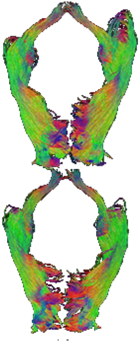
Pathway impairment could be an underlying factor of abnormal behavior and mental illness.

CONNECTIVITY ! BRAIN DYNAMICS ! BEHAVIOR

Brain connectome (from DTI)

Dynamic signals (from fMRI)

Outward symptoms (observed)



USC's Mark and Mary Stevens Neuroimaging and Informatics Institute

Study objectives

Conceive a simplified mathematical object to capture the relationship between brain connectivity and brain function.

Use the geometry of this object to understand the ties between connectivity and function.

Use it to predict brain function and behavior from connectivity information.

Mathematical approach

Graph theory (to interpret the connectome).

Discrete dynamics: repeated iterations of functions (to model the *node* dynamics).

In combination: dynamic networks (or *dynomics*).

The Mandelbrot set

Family of quadratic functions: $f_c(z) = z^2 + c$.

Fix the parameter c in the plane.

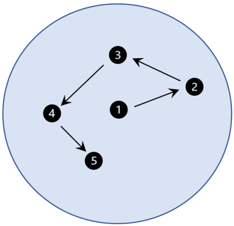
Consider the map $f_c(z) = z^2 + c$.

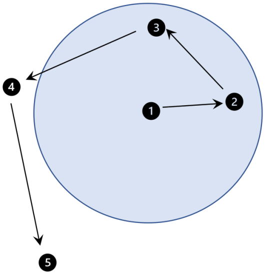
Compute the *orbit* of $z = 0$, by repeatedly applying f_c.

$$0 \;\; ! \;\; f_c(0) \;\; ! \;\; f_c(f_c(0)) \;\; ! \;\; \ldots$$

c is in the Mandelbrot set if this sequence is bounded

c is not in the Mandelbrot set if this sequence $\;!\; \infty$





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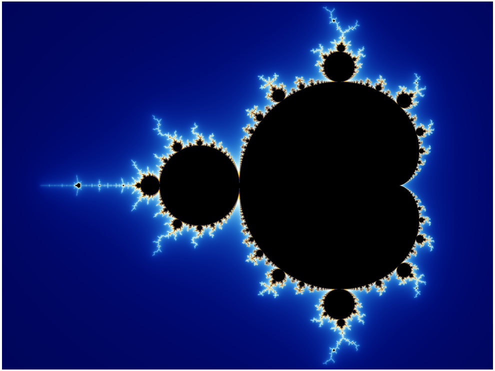


Mandelbrot sets for nodes in a network

3D network

Connectivity matrix

$$\begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}$$

Mandelbrot sets

Defining equations:

$$z_1 \to z_1^2 + c$$

$$z_2 \to (z_1 + z_2)^2 + c$$

$$z_3 \to (z_2 + z_3)^2 + c$$

Mandelbrot sets for nodes in a network

3D network	Connectivity matrix	Mandelbrot sets

$$\begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}$$

Example: $c = -2$ is in the M set for z_1, but not in the M set for z_2.

$z_1: 0 \to 0^2 - 2 = -2 \to (-2)^2 - 2 = 2 \to 2^2 - 2 = 2 \ldots$ (bounded)

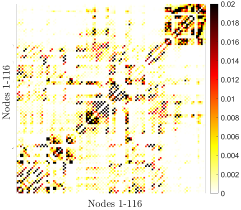
$z_2: 0 \to (0 + 0)^2 - 2 = -2 \to (-2 - 2)^2 - 2 = 14 \to$
$\to (14 + 2)^2 - 2 \ldots$ (unbounded)

How we use mathematics

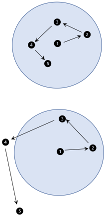
Connectome

Dynamics

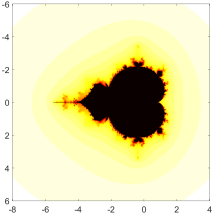
Behavior



Matrix



Discrete dynamics



Mandelbrot sets

Data set used

We tested our method on a set of tractography data for 197 individuals.

The data was collected through the Human Connectome Project.

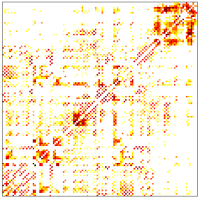
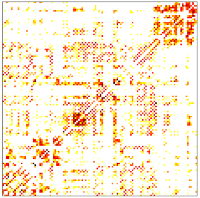
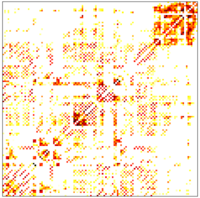
The project aims to identify relationships between connectome patterns and behavior.

Brain dynamics bridge connectome patterns to behavior. We use our dynamical systems approach to illustrate this.

Application

"What is the role of weak connections in synchronization of brain activity?"

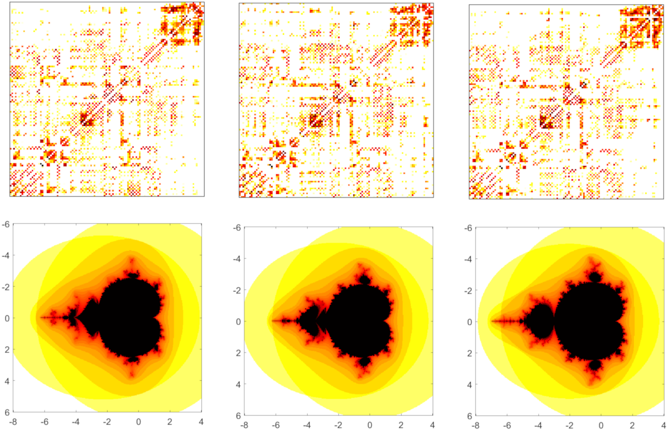
We extracted matrices from DTI connectomes (116 nodes).

Application

For each participant, the corresponding Mandelbrot sets were computed (bottom).

The geometry of the Mandelbrot sets is slightly different between participants, creating a *signature* for each individual.



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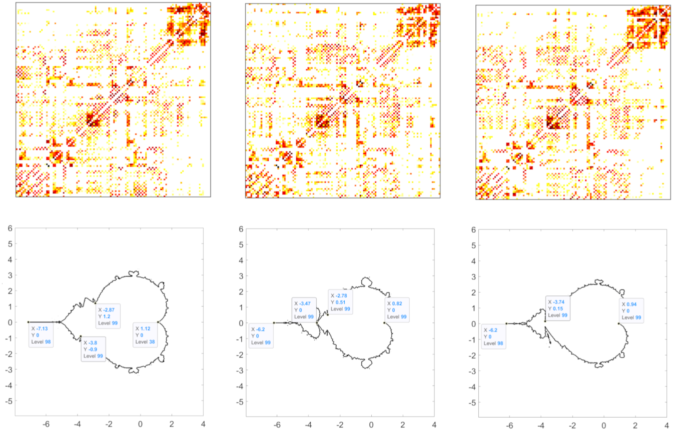
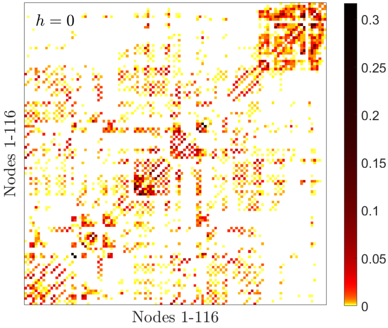


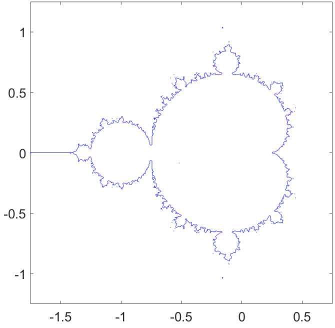
Results on synchronization

Compute Mandelbrot sets separately for each node.

Study when nodes *synchronize* (i.e., have identical M).

"What is the role of weak connections in synchronization of brain activity?"





100% connections

$K = 1$ contours

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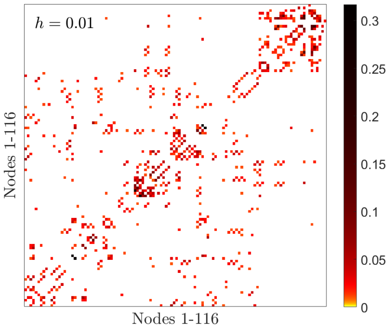
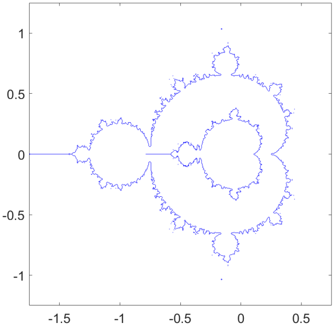




20% connections

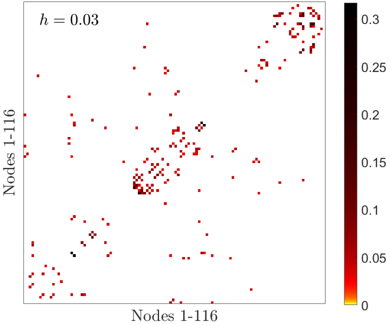
$K = 2$ contours

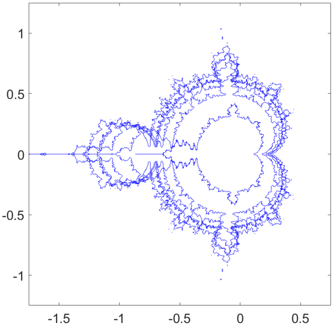
Results on synchronization

Compute Mandelbrot sets separately for each node.

Study when nodes *synchronize* (i.e., have identical M).

"What is the role of weak connections in synchronization of brain activity?"





7% connections

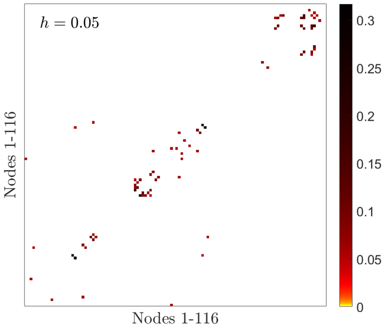
$K = 6$ contours

Results on synchronization

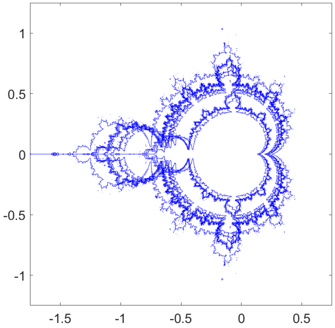
Compute Mandelbrot sets separately for each node.

Study when nodes *synchronize* (i.e., have identical M).

"What is the role of weak connections in synchronization of brain activity?"



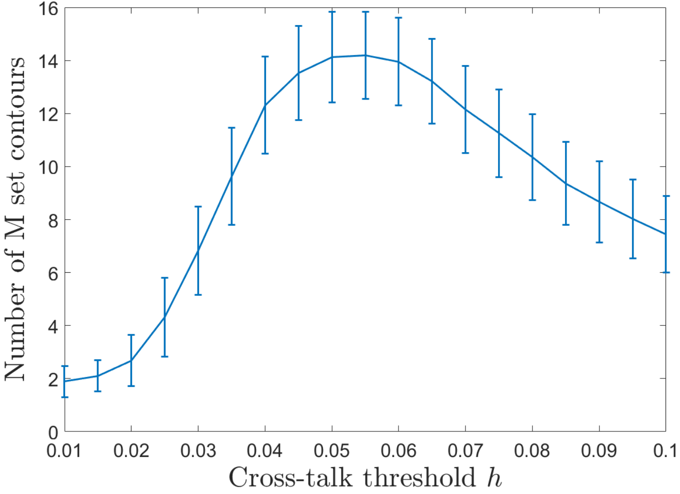
3% connections



$K = 12$ contours

Results on synchronization

Synchronization vs. cross-talk threshold (group statistics)



Conclusions

Weak connections between brain regions play an extremely important role in synchronization.

This has been observed before in networks, but has never been fully explained.

The brain has to constantly switch between “synchronized” and “de-synchronized” patterns.

Our mathematical approach showed that the brain can do this by simply tuning the weak connections.

Acknowledgements

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References

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Thank you!

Questions?